Moral hazard, insurance and some collusion:

a comment on Alger and Ma

Maike Niemann[a] and *Jörg Schiller*[b,*]

May 2005

[a] Deutsche Lufthansa AG, Lufthansa-Basis, 60546 Frankfurt am Main, Germany.
[b] WHU – Otto Beisheim Graduate School of Management, Burgplatz 2, 56179 Vallendar, Germany.
[*] *Corresponding author:* Tel. ++49-261-6509302. E-mail address: joerg.schiller@whu.edu (J. Schiller).

Alger and Ma (2003) presented an interesting model addressing the possibility of collusion between providers and consumers in health insurance markets and the contractual problems arising from the non-observability of the state of nature. There is an error in their proof of Proposition 3, though the proposition is valid. A corrected proof is presented in the next three paragraphs. The location of the error is noted in the concluding paragraph. Readers are assumed to be familiar with the model and its notation.

Proposition 3 characterizes the solution to a maximization. The proof distinguishes two cases: the case

$$\alpha_{\mathrm{h}}^{\tau} - t_{\mathrm{h}}^{\tau} < \alpha_{\mathrm{s}}^{\tau} - m^{\tau} c - t_{\mathrm{s}}^{\tau} , \tag{a}$$

and its opposite case $\alpha_{\mathrm{h}}^{\tau} - t_{\mathrm{h}}^{\tau} \geq \alpha_{\mathrm{s}}^{\tau} - m^{\tau} c - t_{\mathrm{s}}^{\tau}$. The error applies to case (a), which is assumed from here on. The objective function of the maximization is

$$\begin{aligned} &\theta\left[(1-p)U(W - t_{\mathrm{h}}^{\sigma}) + pU(W - \ell + f(m^{\sigma}) - t_{\mathrm{s}}^{\sigma})\right] \\ &+ (1-\theta)\left[(1-p)U(W - t_{\mathrm{h}}^{\tau}) + pU(W - \ell + f(m^{\tau}) - t_{\mathrm{s}}^{\tau})\right] . \end{aligned} \tag{b}$$

The maximands are 10 insurance parameters (m's, α's, t's). The constraints are

$$\alpha_{\mathrm{h}}^{\tau} \geq 0 , \qquad \alpha_{\mathrm{s}}^{\sigma} - m^{\sigma} c \geq 0 , \qquad \alpha_{\mathrm{s}}^{\tau} - m^{\tau} c \geq 0 , \tag{c}$$

$$\theta(t_{\mathrm{s}}^{\sigma} - \alpha_{\mathrm{s}}^{\sigma}) + (1-\theta)[(1-p)(t_{\mathrm{h}}^{\tau} - \alpha_{\mathrm{h}}^{\tau}) + p(t_{\mathrm{s}}^{\tau} - \alpha_{\mathrm{s}}^{\tau})] \geq 0 , \tag{d}$$

$$(1-p)\alpha_{\mathrm{h}}^{\tau} + p(\alpha_{\mathrm{s}}^{\tau} - m^{\tau} c) \geq (1-p)\max[\alpha_{\mathrm{h}}^{\sigma}, 0] + p(\alpha_{\mathrm{s}}^{\sigma} - m^{\sigma} c) , \tag{e}$$

$$\alpha_{\mathrm{s}}^{\sigma} - m^{\sigma} c + (1-p)(t_{\mathrm{h}}^{\sigma} - t_{\mathrm{s}}^{\sigma}) \geq \alpha_{\mathrm{s}}^{\tau} - m^{\tau} c + (1-p)(t_{\mathrm{h}}^{\tau} - t_{\mathrm{s}}^{\tau}) . \tag{f}$$

(b)-(f) reproduce (17) - (22) of Alger and Ma, except that (f) simplifies (22) using (a).

A main step in the proof is to show that the second and third constraint in (c) hold with equality at an optimum. To show that the third constraint in (c) must hold with equality, Alger and Ma in effect argue that, if $\alpha_{\mathrm{s}}^{\tau} - m^{\tau} c > 0$ held for some feasible values of the insurance parameters, then, for small enough positive ε, the all-else-equal parameter variations

$$\alpha_{\mathrm{s}}^{\tau} \to \alpha_{\mathrm{s}}^{\tau} - \varepsilon , \qquad \alpha_{\mathrm{h}}^{\tau} \to \alpha_{\mathrm{h}}^{\tau} + \frac{p}{1-p}\varepsilon , \qquad t_{\mathrm{h}}^{\sigma} \to t_{\mathrm{h}}^{\sigma} - \frac{\varepsilon}{1-p}$$

would increase the objective function (b) and would satisfy the conditions (a) and (c)-(f). Thus, $\alpha_{\mathrm{s}}^{\tau} - m^{\tau} c > 0$ cannot hold at an optimum, and we may assume from here on that

$$\alpha_{\mathrm{s}}^{\tau} - m^{\tau} c = 0 . \qquad \text{(g)}$$

To show in similar fashion that the second constraint in (c) must hold with equality at an optimum, suppose $\alpha_{\mathrm{s}}^{\sigma} - m^{\sigma} c > 0$ held. Then, for small enough positive ε, the all-else-equal parameter variations

$$\alpha_{\mathrm{s}}^{\sigma} \to \alpha_{\mathrm{s}}^{\sigma} - \varepsilon , \quad t_{\mathrm{s}}^{\sigma} \to t_{\mathrm{s}}^{\sigma} - \varepsilon , \quad \alpha_{\mathrm{h}}^{\tau} \to \alpha_{\mathrm{h}}^{\tau} - \frac{p}{1-p} \varepsilon , \quad t_{\mathrm{h}}^{\tau} \to t_{\mathrm{h}}^{\tau} - \frac{p}{1-p} \varepsilon \qquad \text{(h)}$$

would increase the objective function (b) and would satisfy the conditions (a) and (c)-(f). To verify that the first constraint in (c) is satisfied after the variations, first simplify (e) using (g), and note that the simplified (e), together with the hypothesis $\alpha_{\mathrm{s}}^{\sigma} - m^{\sigma} c > 0$, implies $\alpha_{\mathrm{h}}^{\tau} > 0$ before the variations, which makes room for the decrease in $\alpha_{\mathrm{h}}^{\tau}$ imposed by the variations. Since the variations would lead to an improvement in objective function, the hypothesis $\alpha_{\mathrm{s}}^{\sigma} - m^{\sigma} c > 0$ cannot hold at an optimum. Instead $\alpha_{\mathrm{s}}^{\sigma} - m^{\sigma} c = 0$ must hold.

The published proof used parameter variations different from (h), and these different variations did not satisfy constraint (f). That was the error. It occurs in the first 14 lines of Appendix Section A.3 in the published paper, ending with the words “has been decreased.” The slip may be corrected by replacing the 14 lines with the preceding three paragraphs.

References

Alger, I., Ma, C.-T.A., 2003. Moral hazard, insurance, and some collusion. Journal of Economic Behavior and Organization 50, 225-247.